LESSON 7.1

Similar Polygons

Objectives

7.1 Similar Polygons
- a. I can define similar polygons.
- b. I can write similarity statements.
- c. I can determine the scale factor of two similar polygons.
- d. Given similar polygons, I can use proportions to write and solve equations.

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Lesson 7.1: S

Pick up a handout - protractor should be on your desk

LESSON 7.1

Similar Polygons

You know that figures that have the same shape and size are congruent figures. Figures that have the same shape but not necessarily the same size are **similar** figures. To say that two figures have the same shape but not necessarily the same size is not, however, a precise definition of similarity.

Is your reflection in a fun-house mirror similar to a regular photograph of you? The images have a lot of features in common, but they are not mathematically similar. In mathematics, you can think of similar shapes as dilations (enlargements or reductions) of each other with no irregular distortions.

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Lesson 7.1: Similar Polygons

Warm-up: Dilate $\triangle$**ABC with a scale factor of 2, where the center of dilation is the origin (0, 0).**
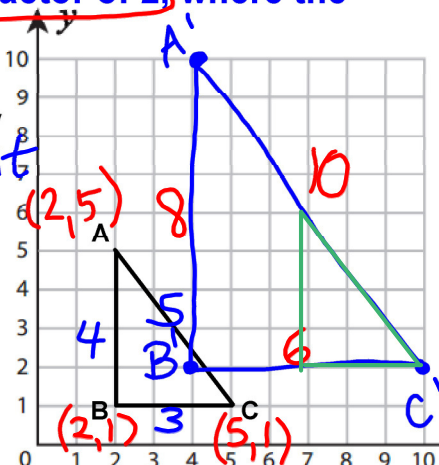
The dilation results in two similar triangles

$\triangle ABC \sim \triangle A'B'C'$ means $\triangle ABC$ is similar to $\triangle A'B'C'$

Is this a reduction or enlargement? *enlargement*

Hint: Use Pythagorean Thoerem to find the Hypotenuse lengths.

Pre-image: AB __4__   BC __3__   CA __5__

Image: A'B' __8__   B'C' __6__   C'A' __10__

Ratio of corresponding side lengths

$\dfrac{A'B'}{AB} = \dfrac{8}{4} = 2$ $\dfrac{B'C'}{BC} = \dfrac{6}{3} = 2$ $\dfrac{C'A'}{CA} = \dfrac{10}{5}$

$\dfrac{AB}{A'B'} = \dfrac{4}{8} = \dfrac{1}{2}$ $\dfrac{BC}{B'C'} = \dfrac{3}{6} = \dfrac{1}{2}$ $\dfrac{CA}{C'A'} = \dfrac{5}{10} = \dfrac{1}{2}$

$4^2 + 3^2 = (CA)^2$ $6^2 + 8^2 = (C'A')^2$

$16 + 9 = (CA)^2$ $36 + 64 = ?$

$\sqrt{25} = \sqrt{(CA)^2}$ $\sqrt{100} = \sqrt{(C'A')^2}$

$5 = CA$ $10 = C'A'$

Coordinates: (2,5) A, (2,1) B, (5,1) C

Now use patty paper to compare the corresponding angles.

How do the ratios of the corresponding sides compare?

They are proportional

How do the corresponding angles compare?
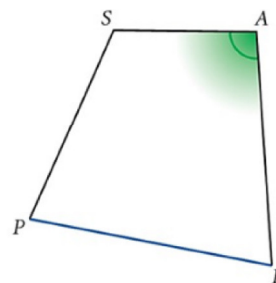
They are Congruent

Two Polygons are Similar if and only if all ratios of <u>corresponding sides</u> are *Proportional* and all pairs of <u>corresponding angles</u> are *Congruent*

The statement CORN~PEAS says that quadrilateral CORN is similar to quadrilateral PEAS. The order of letters tells you which segments and which angles in the two polygons correspond.
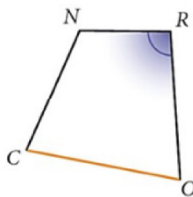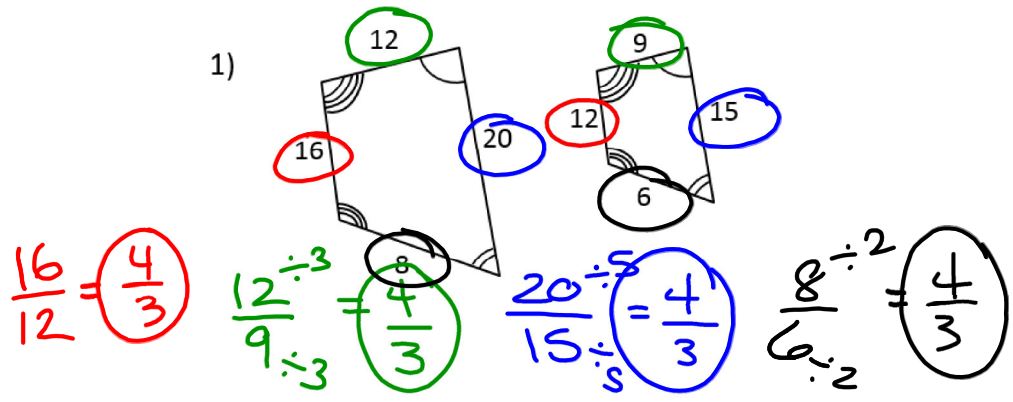
Corresponding angles are congruent:

$\angle C \cong \angle P$ $\angle R \cong \angle A$
$\angle O \cong \angle E$ $\angle N \cong \angle S$

Corresponding segments are proportional:

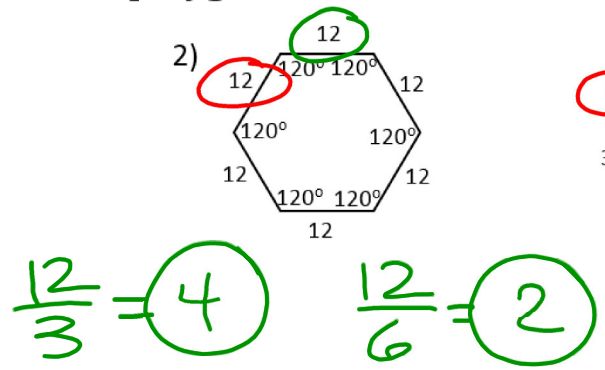$\dfrac{CO}{PE} = \dfrac{OR}{EA} = \dfrac{RN}{AS} = \dfrac{NC}{SP}$

Are the polygons below similar? Explain why or why not.

1)

12	9
16	20	12	15
8	6

$$\frac{16}{12} = \frac{4}{3}$$

$$\frac{12 \div 3}{9 \div 3} = \frac{4}{3}$$

$$\frac{20 \div 5}{15 \div 5} = \frac{4}{3}$$

$$\frac{8 \div 2}{6 \div 2} = \frac{4}{3}$$

Similar / Not Similar (circle one)

Reason: **Ratios are proportional & Corresponding angles are congruent**

Are the polygons below similar? Explain why or why not.

2)

Hexagon 1: sides 12, all angles 120°

Hexagon 2: sides 3 and 6, all angles 120°

$$\frac{12}{3} = 4$$

$$\frac{12}{6} = 2$$

Similar / Not Similar (circle one)

Reason: **Ratios are not proportional**

The following polygons are similar. Write a similarity statement and find the scale factor of figure *A* to figure *B*.
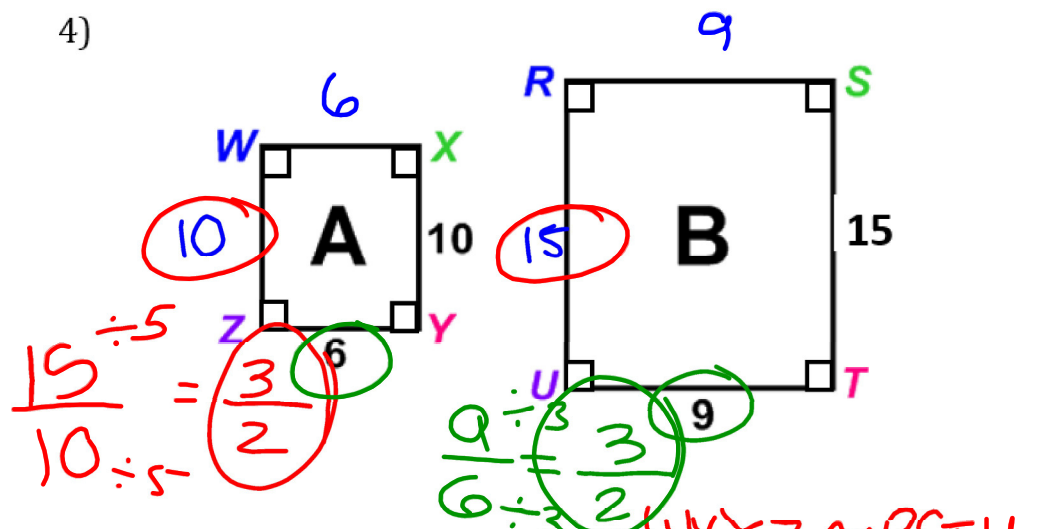
3)

A   18   D   12

Original   *pre-image*   *image*

9   **A**   B   6   **B**

12   8

C

B on Top → $\frac{6}{9} = \frac{2}{3}$ $\frac{12}{18} = \frac{2}{3}$ $\frac{8}{12} = \frac{2}{3}$
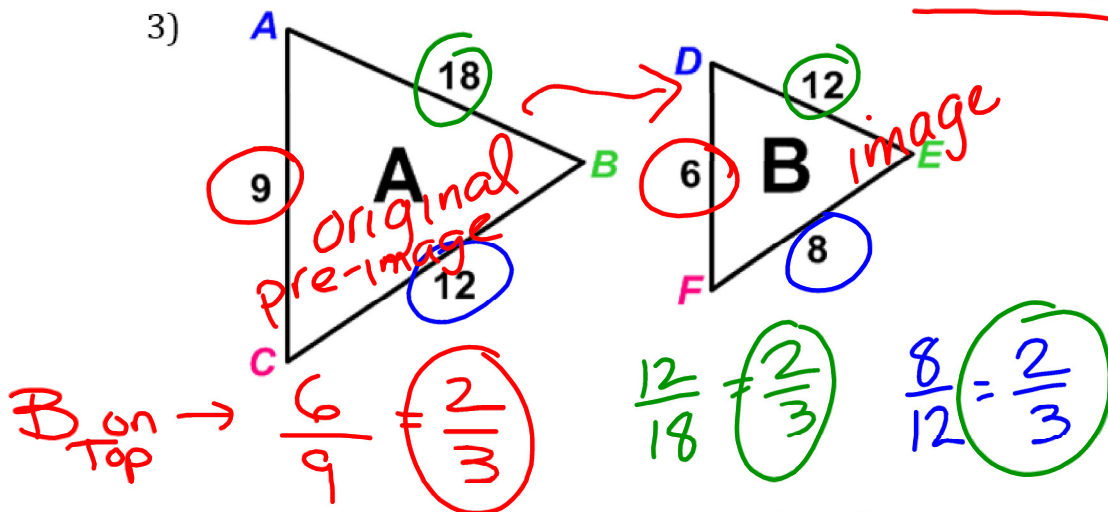
Similarity Statement: $\triangle ABC \sim \triangle DEF$

Scale factor of figure B: $\frac{2}{3}$

Reduction / Enlargement? (Circle one)

The following polygons are similar. Write a similarity statement and find the scale factor of figure *A* to figure *B*.

4)

6   9

W   X   R   S

10   **A**   10   15   **B**   15

Z   Y   U   T

6   9

$\frac{15 \div 5}{10 \div 5} = \frac{3}{2}$ $\frac{9 \div 3}{6 \div 3} = \frac{3}{2}$
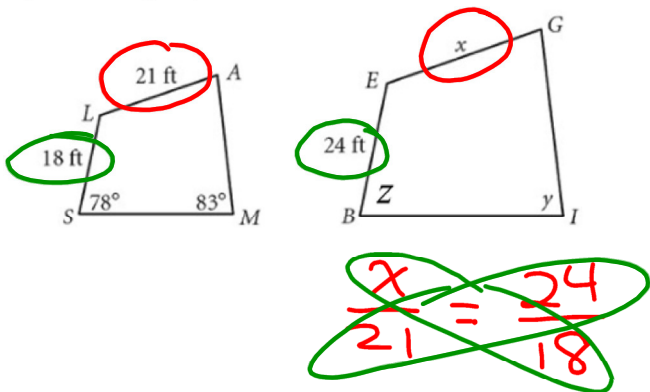
Similarity Statement: $WXYZ \sim RSTU$

Scale factor of figure A to figure B: $\frac{3}{2}$

Reduction / **Enlargement?** (Circle one)

5) In the diagram, SMAL~BIGE.

21 ft A
L
18 ft
78°   83° M
S

E
x
G
24 ft
Z
B   y   I

$x = \underline{28}$
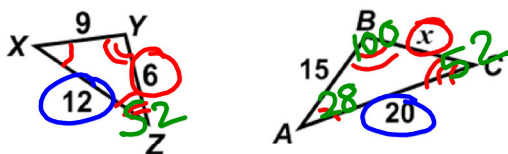$\rightarrow y = \underline{83}$
$\rightarrow z = \underline{78}$

$$\frac{x}{21} = \frac{24}{18}$$

$18x = (21)(24)$

$\frac{18x}{18} = \frac{504}{18}$

$x = 28$

6) In the diagram, $\triangle XYZ \sim \triangle ABC$.

9 Y
X
6
12
Z

B
100 x
15
28   20
A
C

Scale factor of $\triangle XYZ$ to $\triangle ABC$: $\frac{5}{3}$

Reduction / Enlargement? (Circle one)

$x = \underline{10}$

If $m\angle A = 28°$ and $m\angle B = 100°$, what is $m\angle Z$?

$\underline{52°}$

Scale factor : $\frac{image}{Pre\text{-}image} = \frac{20 \div 4}{12 \div 4} = \frac{5}{3}$
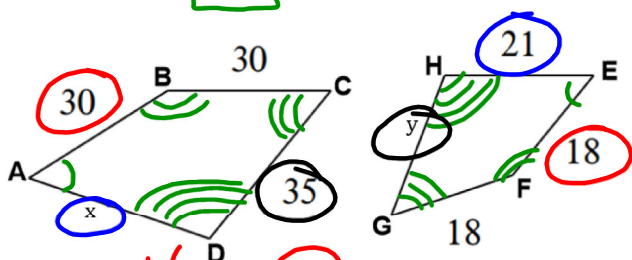
$$\frac{x}{6} = \frac{20}{12}$$

$12x = (6)(20)$

$\frac{12x}{12} = \frac{120}{12}$

$6\left(\frac{5}{3}\right) = 10$

Pre-image   Scale factor   image

7) In the diagram, $ABCD \sim EFGH$.

30

B   30   C
(30)
H   (21)   E

A   35   y   (18)

x   F

D   G   18

Scale factor of $ABCD$ to $EFGH$: $\dfrac{3}{5}$

(Reduction) / Enlargement? (circle one)

$x = 35$

$y = 21$

$\dfrac{18 \div 6}{30 \div 6} = \dfrac{3}{5}$

$\dfrac{21}{x} = \dfrac{18}{30}$

$\dfrac{18x}{18} = \dfrac{630}{18}$

$\dfrac{y}{35} = \dfrac{18}{30}$

$\dfrac{30y}{30} = \dfrac{630}{30}$

Homework: 7.1 Similar Polygons Homework