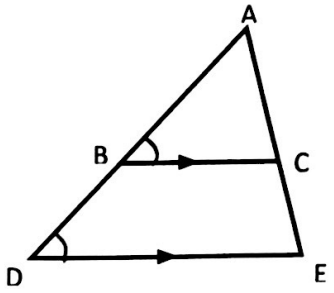
7.5 Proportional Segments Between Parallel Lines
Geometry 3313

Ratio Researcher: Key
Date: _____ **Period:** _____

Given: $\overline{BC} \parallel \overline{DE}$

Prove: $\Delta ABC \sim \Delta ADE$



BC ∥ DE
Given

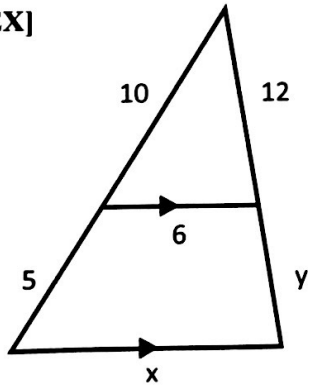
∠ABC ≅ ∠ADE
Corresponding ∠ Postulate

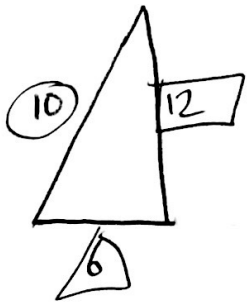
∠A ≅ ∠A
Reflexive Prop

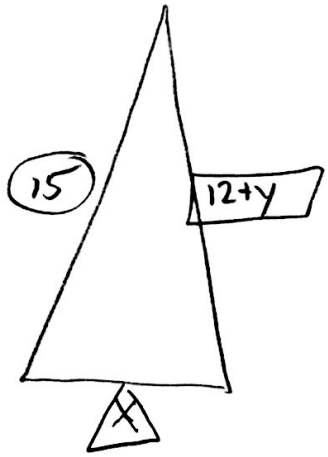
ΔABC ~ ΔADE
AA ~

We have just shown that a line parallel to one side of a triangle intersecting the other two sides will create similar triangles. In cases like this, when we are asked to set up proportions to find missing lengths, it may be beneficial to draw the triangles separately.

EX]







$\frac{10}{15} = \frac{6}{x}$

$10x = 90$

$x = 9$

$\frac{10}{15} = \frac{12}{12+y}$

$120 + 10y = 180$

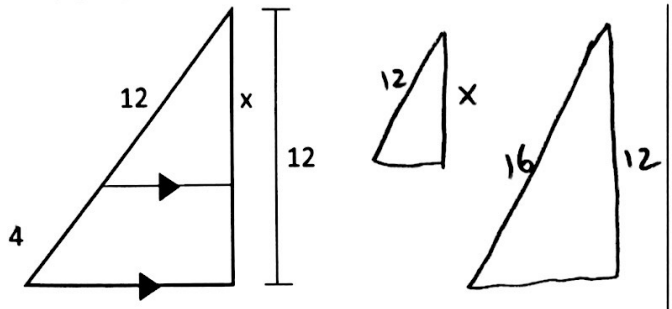
$10y = 60$

$y = 6$

X = 9

Y = 6

Set up proportions to solve for the missing variables. You may find it beneficial to separate the triangles.

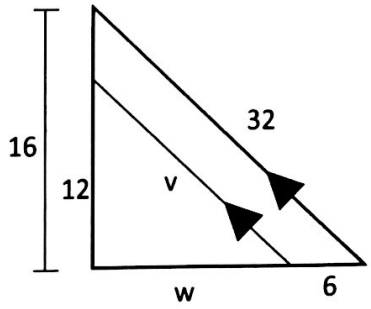


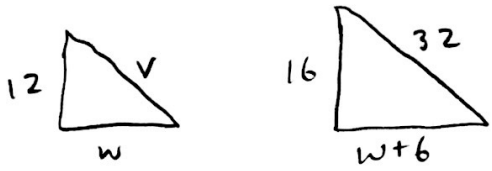
$\frac{x}{12} = \frac{12}{16}$

$16x = 144$

$x = 9$

x = 9





$\frac{w}{w+6} = \frac{12}{16}$

$16w = 12w + 72$

$4w = 72$

$w = 18$

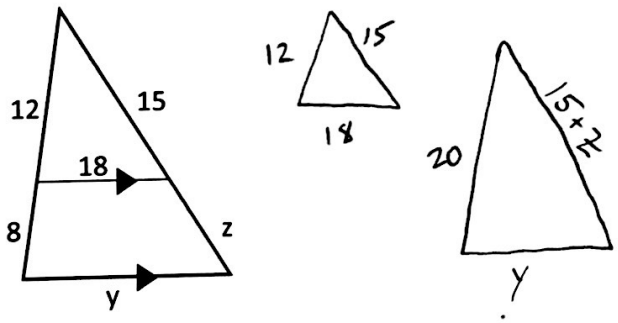
$\frac{v}{32} = \frac{12}{16}$

$16v = 384$

$v = 24$

w = 18

v = 24



$\frac{20}{12} = \frac{15+z}{15}$

$300 = 180 + 12z$

$120 = 12z$

$10 = z$

$\frac{20}{12} = \frac{y}{18}$

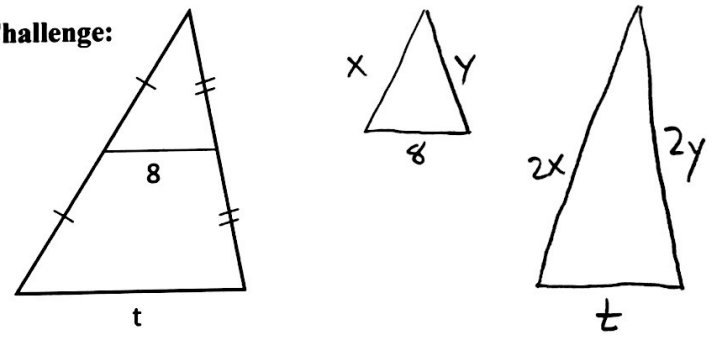
$360 = 12y$

$30 = y$

z = 10

y = 30

Challenge:



$\frac{2x}{x} = \frac{t}{8}$

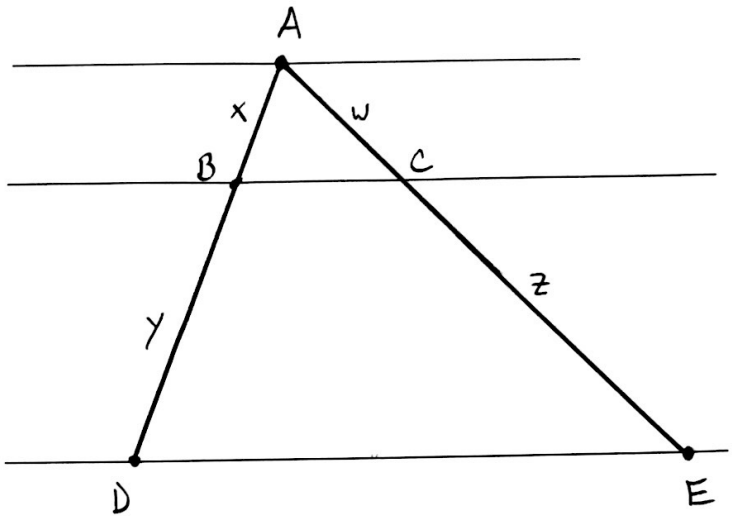
$\frac{2}{1} = \frac{t}{8}$

$16 = t$

t = 16

The three line segments below are all parallel to each other. Follow the directions, in the order they are written.

1. **Put point A on the top line (anywhere).**
2. **Put point D on the bottom line.**
3. **Put point E on the bottom line, away from D, in a place such that ΔADE will NOT be isosceles.**
4. **Use your straight edge to draw $\overline{AD}$ and $\overline{AE}$**
5. **Place point B at the intersection of $\overline{AD}$ and the middle segment.**
6. **Place point C at the intersection of $\overline{AE}$ and the middle segment.**
7. **Place an "x" between A and B.**
   Place a "y" between B and D.
   Place a "w" between A and C.
   Place a "z" between C and E.
7. **Measure each of the following lengths**
8. **Calculate the following ratios (round if necessary).**



x = 17 mm w = 23 mm

y = 40 mm z = 53 mm

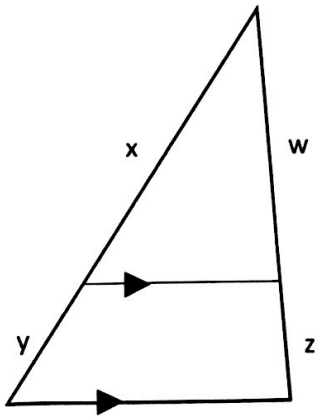
Calculate the following ratios.

$\frac{x}{y} = \frac{17}{40} = 0.425$ $\frac{w}{z} = \frac{23}{53} \approx 0.434$

Compare your results with your groupmates. You should all have different triangles, different measurements, and different ratios, but is there anything you notice among your answers?

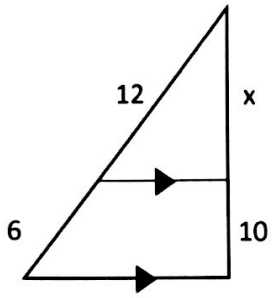
The ratios of $\frac{x}{y}$ and $\frac{w}{z}$ are approximately equal.

When a line parallel to one side of a triangle intersects the other two sides of the triangle, it will divide those two sides proportionally; that is: we can set up a proportion for the four pieces that are created. This is basically a shortcut for a VERY SPECIAL CASE.



★ $\frac{x}{y} = \frac{w}{z}$ ★

Set up proportions to solve for the missing variables.

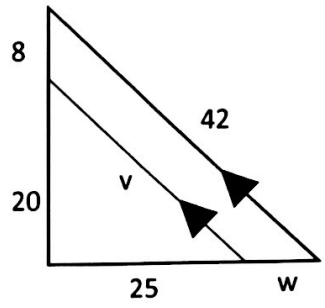


$\frac{12}{6} = \frac{x}{10}$

$120 = 6x$

$20 = x$

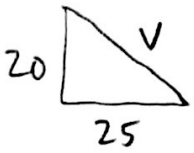
x = 20

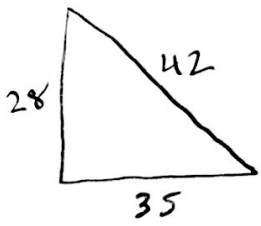


$\frac{20}{8} = \frac{25}{w}$

$20w = 200$

$w = 10$





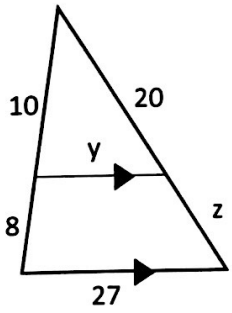
$\frac{20}{28} = \frac{v}{42}$

$28v = 840$

$v = 30$

w = 10

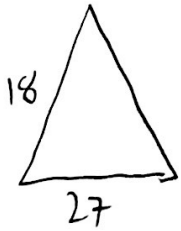
v = 30



$\frac{10}{8} = \frac{20}{z}$

$10z = 160$

$z = 16$



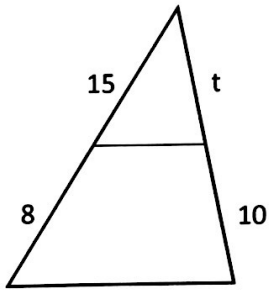
$\frac{10}{18} = \frac{y}{27}$

$18y = 270$

$y = 15$

z = 16

y = 15



$\frac{15}{8} = \frac{t}{10}$

$150 = 8t$

$18.75 = t$

t = 18.75