**Regular Polygon:** a polygon that is equiangular (all angles are equal in measure) and equilateral (all sides have the same length).

Suppose the following two pizzas cost the same price. Which pizza is the better deal, a circular pizza with a 16 inch diameter or a regular hexagonal pizza where both the radius and side length 8 inches long? Why?
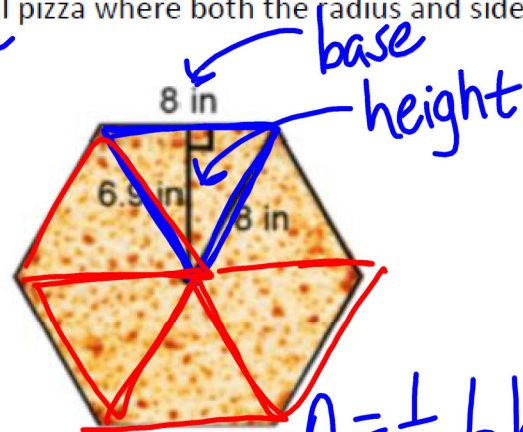
base
height

8 in

6.9 in   8 in

$r=8$

16 in
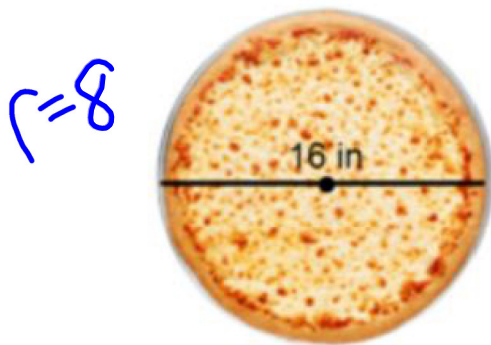
$A = \pi r^2$

$A = \pi (8)^2$

$A = 201.1 \ in^2$

better deal

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(8)(6.9)$

$A = 27.6$

$A = 6(27.6)$

$A = 165.6 \ in^2$

**Investigation #2: Deriving the Area of a Regular Polygon**

Step 1: Consider a regular pentagon with side length s, divided into congruent isosceles triangles. Each triangle has a base s and a height a. Step 1 What is the area of one isosceles triangle in terms of a and s?
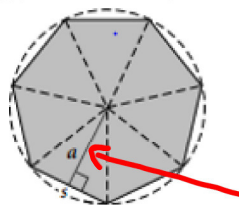
apothem

Regular pentagon

Step 2: What is the area of this pentagon in terms of a and s?

Step 3: Repeat Steps 1 and 2 with other regular polygons and complete the table below.
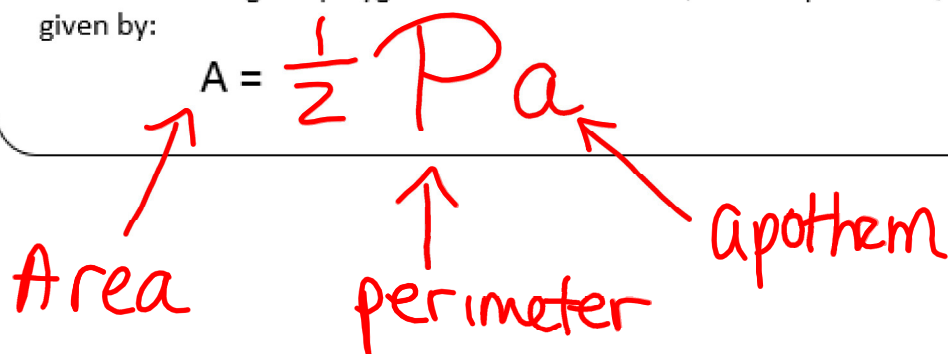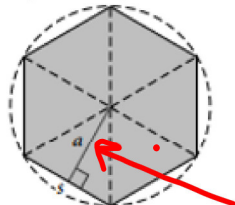
Regular hexagon   Regular heptagon
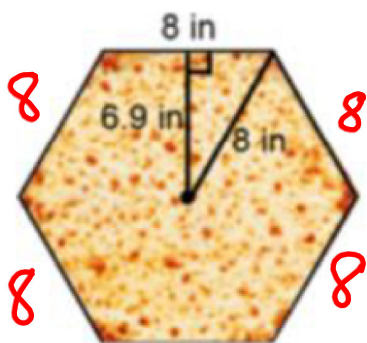
Area Formula for Regular Polygons:

The area of a regular polygon where A is the area, P is the perimeter, a is the apothem, is given by:

$$A = \frac{1}{2} P a$$
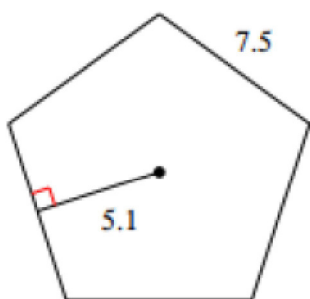
Area

perimeter

apothem

1.   Use the area formula for a regular polygon to find the area of the pizza below.

8 in

8

6.9 in

8 in

8

8

8

8

$$A = \frac{1}{2} P a$$

$$A = \frac{1}{2} (48)(6.9)$$

$$A = 165.6 \text{ in}^2$$

$$P = 6(8)$$

$$P = 48$$

2.   Find the area of the regular pentagon below.
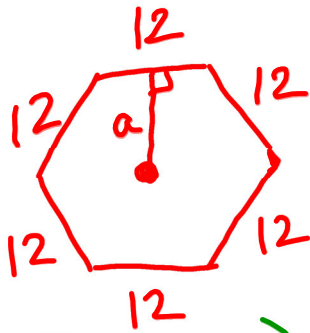
7.5

5.1

↖ 5 sides

$$A = \frac{1}{2} P a$$

$$A = \frac{1}{2}(37.5)(5.1)$$

$$\boxed{A = 95.625}$$

$$P = 5(7.5)$$

$$P = 37.5$$

3. Suppose a regular hexagon has an area of 522 m² and a side length of 12 m. Find the length of the apothem of the hexagon.
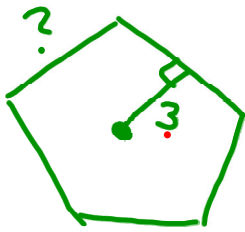
12

6 sides

$A = \frac{1}{2}Pa$

$522 = \frac{1}{2}(72)a$

$522 = 36a$

$\frac{522}{36} = \frac{36a}{36}$

$P = 6(12) = 72$

$\boxed{a = 14.5\,m}$

4. Suppose a regular pentagon has an area of 33 ft² and an apothem length of 3 ft. Find the length of each side of the pentagon.

5 sides

$A = \frac{1}{2}Pa$

$33 = \frac{1}{2}P(3)$

$P = n \cdot s$

$\frac{33}{1.5} = \frac{1.5\,P}{1.5}$

$\frac{22}{5} = \frac{5s}{5}$

$22 = P$

$\boxed{4.4\ ft}$

Assignment:

8.3 Regular Polygons. #1-3 only

Note for #2a - side length is 12